

University of California at Santa Barbara

Physics Department

Physics 225A Homework 4

Read Halzen and Martin sections 2.12 and 2.14. These were not covered in class, but they are straightforward.

Problems 7 and 8 are optional. Do them if you feel like it.

- **Problem 1**

Imagine that quarks had integral spin.

(a) Predict the pattern J^P of quantum numbers for $\sigma_i\sigma_j$, where σ_i is a hypothetical constituent particle with $J^P = 0^+$. How does this compare with the quantum numbers for the actual mesons?

(b) Predict the pattern of J^P for $\sigma_i v_j$, where v_j are particles with $J^P = 1^-$. Compare with actual mesons.

- **Problem 2**

Assume that there were stable diquarks¹ (for example, if there were two colors).

(a) What SU(3) flavor-multiplets would exist?

(b) What isospin multiplets would comprise each SU(3) multiplet?

(c) What would be the J^P of the lowest mass states?

- **Problem 3**

Halzen and Martin, Problem 2.12.

Hint: You may want to use equations 2.60 and 2.62 in Halzen and Martin. Also, remember that you can operate with $I_{\pm}$, $U_{\pm}$ and $V_{\pm}$ to move from one state in the multiplet to the next.

- **Problem 4**

Halzen and Martin, Problem 2.17. Express your answers in terms of μ_u , μ_d , and μ_s instead of μ_p .

Hint: the Σ^+ wavefunction, for example, can be obtained from the p wavefunction by replacing the d quark with the s quark.

¹a diquark is a qq bound state

Then assume perfect SU(3) symmetry ($m_u = m_d = m_s$), and using the measured value of the proton magnetic moment predict the magnetic moments of all other baryons in the multiplet. Compare with the experimental values.

- **Problem 5**

Halzen and Martin, Problem 2.18.

- **Problem 6**

Halzen and Martin, Problem 2.30.

Note: Halzen and Martin refers to the F meson. The more modern name for this meson is D_s . This is the $c\bar{s}$ meson.

- **Problem 7**

Halzen and Martin, Problem 2.31.

- **Problem 8**

Halzen and Martin, Problem 2.32.

- **Problem 9**

Halzen and Martin, Problem 2.22.

- **Problem 10**

Discuss the possible decay modes of the Ω^- -hyperon allowed by conservation laws, and show that weak decay is the only possibility.

- **Problem 11**

Isospin arguments can in some cases be used to relate amplitudes from weak processes, even if isospin is not conserved in weak interactions. Here is an example which is important for what today is a very hot topic in high energy physics.

Consider the (weak) decay of a B meson into two pions. The possible Feynman diagrams are shown in Figure 1. The diagrams on the left are called *tree* diagrams, the diagrams on the right are called *penguin* diagrams.

It turns out that in the absence of penguins, a measurement of the difference in rates between $B^0 \rightarrow \pi^+\pi^-$ and $\bar{B}^0 \rightarrow \pi^+\pi^-$ would yield some

However, isospin symmetry can come to the rescue. Let's explore how this works.

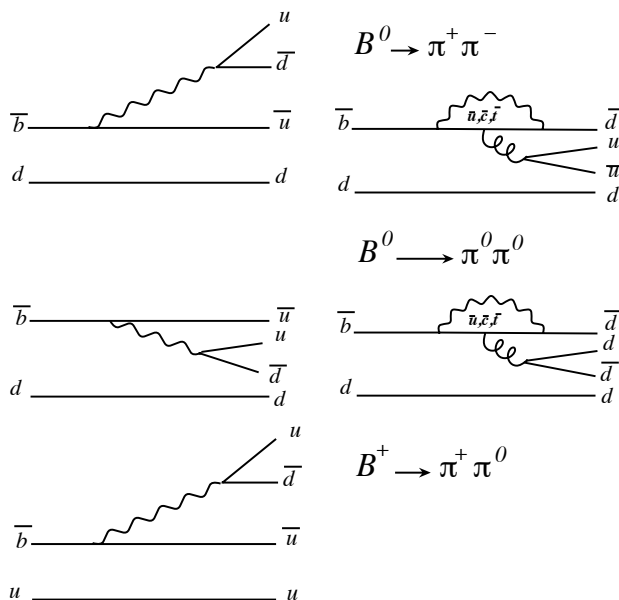
$$B^0 = |\bar{b}d\rangle, B^+ = |\bar{b}u\rangle, \bar{B}^0 = |b\bar{d}\rangle, B^- = |b\bar{u}\rangle).$$


Figure 1: Feynman diagrams for $B \rightarrow \pi\pi$

The weak process does not conserve isospin. The isospin of the initial state is $I=1/2$. The isospin of the final state is $I=0$ or $I=2$. (Pions have $I=1$; two pions can have $I=0,1,2$; the B has $J=0$, so the two pion state has $L=0$; Bose statistics then forbids $I=1$ in this case). Thus, there are two pieces of the amplitude, one with $\Delta I=1/2$ and the other one with $\Delta I=3/2$.

The tree diagrams are a mixture of $\Delta I=1/2$ and $\Delta I=3/2$, because there is a $\Delta I=1/2$ transition in $\bar{b} \rightarrow \bar{u}$, plus the virtual W carries away $\Delta I=1$ (it couples to the pion!). On the other hand the penguin diagrams are purely $\Delta I=1/2$ because the gluon has $I=0$ (roughly speaking it couples to $\bar{u}u + \bar{d}d$).

[Note: you may think that you should be able to write a penguin diagram also for the $B^+ \rightarrow \pi^+\pi^0$; in fact you should be able to write two of these, one with $g \rightarrow u\bar{u}$ and one with $g \rightarrow d\bar{d}$ (try it!). However, because the $\pi^+\pi^0$ state has $I=2$, and the penguins are $\Delta I=1/2$, they do not contribute.]

Armed with all of this information, show that

$$\frac{1}{\sqrt{2}}A^{+-} + A^{00} = A^{+0}$$

where A^{+-} , A^{00} , and A^{+0} are the amplitudes for B decays into two charged pions, two neutral pions, and one charged + one neutral pion respectively.

Hint: one way to apply isospin symmetry concepts to, for example, a $\Delta I=1/2$ decay, is by postulating that a hypothetical particle of $I=1/2$ (aka a *spurion*) is added to the initial state, and then treating the decay as an isospin-conserving process.

Using identical arguments, one can derive the same relationship for the charged conjugate amplitudes, i.e., the amplitudes for the B^0 and B^- decays. Then, it turns out that if all of these amplitudes are measured, the problem of penguin pollution can be overcome. We will see how this works and what this means next quarter.